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| **Course Code:** CT2352 | **Course Name:** Lab - Design & Analysis of Algorithms |

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| **Name of the Student:**  S Akshansh | **Sem/Sec:**  6th/A |
| **Branch:**  Computer Technology | **Roll no.:** 72  **Enrolment no.:** 19010927 |

Practical – 3

**Aim –** Solving Homogenous Recurrences.

**Theory –**

A **recurrence relation** is an [equation](https://en.wikipedia.org/wiki/Equation) that [recursively](https://en.wikipedia.org/wiki/Recursion) defines

a [sequence](https://en.wikipedia.org/wiki/Sequence) or multidimensional array of values, once one or more initial terms are given; each further term of the sequence or array is defined as a [function](https://en.wikipedia.org/wiki/Function_(mathematics)) of the preceding terms. Below are the steps required to solve a recurrence equation using the polynomial reduction method:

1. Form a characteristic equation for the given recurrence equation.
2. Solve the characteristic equation and find the roots of the characteristic equation.
3. Simplify the solution with unknown coefficients.
4. Solve the equation with respect to the initial conditions to get a specific solution.

# Example:

Consider the following second-order recurrence equation:

## T(n) = a1T(n-1) + a2T(n-2)

For solving this equation formulate it into a characteristic equation. Let us rearrange the equation as follows:

## T(n) - a1T(n-1) - a2T(n-2) = 0

Let, T(n) = xn

Now we can say that T(n-1) = xn-1 and T(n-2)=xn-2

Now the equation will be:

xn + a1xn-1 + a2xn-2 = 0

After dividing the whole equation by xn-2 [since x is not equal to 0] we get:

## x2 + a1x + a2 = 0

We have got our characteristic equation as x2+a1x+a2=0

Now, find the roots of this equation.

Three cases may exist while finding the roots of the characteristic equation and those are:

# Case 1: Roots of the Characteristic Equation are Real and Distinct

If there are r number of distinct roots for the characteristic equation then it can be said that r number of fundamental solutions are possible. One can take any linear combination of roots to get the general solution of a linear recurrence equation.

If r1, r2, r3……, rk are the roots of the characteristic equation then the general solution of the recurrence equation will be:

tn = c1r1n + c2r2n + c3r3n +...........+ ckrkn

# Case 2: Roots of the Characteristic Equation are Real but not Distinct

Let us consider the roots of the characteristic equations are not distinct and the roots r is in the multiplicity of m. In this case, the solution of the characteristic equation will be:

r1 = rn r2 = nrn

r3 = n2rn

## .........

rm = nm-1rn

Therefore, include all the solutions to get a general solution of the given recurrence equation.

# Case 3: Roots of the Characteristic Equation are Distinct but Not Real

If the roots of the characteristic equation are complex, then find the conjugate pair of roots.

If r1 and r2 are the two roots of a characteristic equation, and they are in conjugate pair with each other it can be expressed as:

## r1 = reix r2 = re-ix

The general solution will be:

tn = rn(c1cos nx + c2sin nx)

# Example:

Let’s solve the given recurrence relation: T(n) = 7\*T(n-1) - 12\*T(n-2)

Let T(n) = xn

Now we can say that T(n-1) = xn-1 and T(n-2)=xn-2 And dividing the whole equation by xn-2, we get: x2 - 7\*x + 12 = 0

Below is the implementation to solve the given quadratic equation: R1 = 4 & R2 = 3

**Code –**

#include<iostream>

using namespace std;

void Roots(float a, float b, float c)

{

    float d,x1,x2,real,imag;

    d = (b\*b) - (4\*a\*c);

    if(d>0){

        x1 = ((-b) + sqrt(d)) / (2\*a);

        x2 = ((-b) - sqrt(d)) / (2\*a); cout<<"x1 = "<<x1<<endl; cout<<"x2 = "<<x2<<endl;

    }

    else if(d == 0){

        x1 = x2 = (-b)/(2\*a); cout<<"x1 = "<<x1<<endl; cout<<"x2 = "<<x2<<endl;

    }

    else if(d<0){

        real = (-b)/(2\*a);

        imag = (sqrt(-d) / (2\*a));

        cout<<"x1 = "<<real<<" + "<<imag<<"i"<<endl; cout<<"x2 = "<<real<<" - "<<imag<<"i"<<endl;

    }

    cout<<"General Solution - ";

    cout<<"C1("<<x1<<")^n + "<<"C2("<<x2<<")^n"<<endl;

}

int main()

{

    float a,b,c;

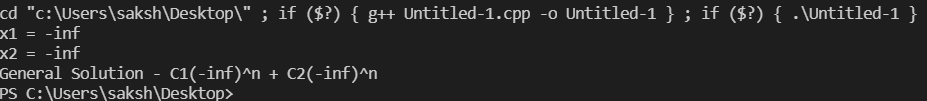
    cin>>a>>b>>c;

    Roots(a,b,c);

    return 0;

}

**Output –**

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**Conclusion –** I have successfully executed the code on homogenous recurrences.